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On the Diophantine equation $x^2 + 2^\alpha 5^\beta 17^\gamma = y^n$

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Abstract. In this paper, we find all solutions of the Diophantine equation $x^2 + 2^\alpha 5^\beta 17^\gamma = y^n$ in positive integers $x, y \geq 1$, $\alpha, \beta, \gamma, n \geq 3$ with $\gcd(x, y) = 1$.

1 Introduction

There are many results concerning the generalized Ramanujan-Nagell equation

$$x^2 + C = y^n, \quad (1)$$

where $C > 0$ is a given integer and x, y, n are positive integer unknowns with $n \geq 3$. Results obtained for general superelliptic equations clearly provide effective finiteness results for this equation, too (see for example [8], [31], [32] and the references given there). The first result concerning the above equation was due to V. A. Lebesgue [23] and it goes back to 1850, where he proved that the above equation has no solutions for $C = 1$. More recently, other values of C were considered. Tengely [33] solved the equation with $C = b^2$, b odd and $3 \leq b \leq 501$. The case where $C = p^k$, a power of a prime number, was studied in [5], [21], [20] for $p = 2$, in [6], [4], [24] for $p = 3$, in [1], [2] for $p = 5$, and in [27] for $p = 7$. The case $C = p^{2k}$ with $2 \leq p < 100$ prime and $\gcd(x, y) = 1$ was solved by Bérczes and Pink [9]. For arbitrary primes, some advances can be found in [7]. In [13], the cases with $1 \leq C \leq 100$ were completely solved. The solutions for the cases $C = 2^a \cdot 3^b$, $C = 2^a \cdot 5^b$ and $C = 5^a \cdot 13^b$, when x and y are coprime, can be found in [3], [25], [26], respectively. Recent progress on the subject were made in the cases $C = 5^a \cdot 11^b$, $C = 2^a \cdot 11^b$, $C = 2^a \cdot 3^b \cdot 11^c$, $C = 2^a \cdot 5^b \cdot 13^c$ and can be found in [16], [15], [14], [18]. For related results concerning equation (1) see [10], [22], [29], [30] and the references given there. For a survey concerning equation (1) see [12].

In this paper, we are interested in solving the Diophantine equation

$$x^2 + 2^\alpha 5^\beta 17^\gamma = y^n, \quad \gcd(x, y) = 1, \quad x, y \geq 1, \quad \alpha, \beta, \gamma \geq 0, \quad n \geq 3. \quad (2)$$

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Our result is the following.

Theorem 1. *The equation (2) has no solution except for:*

$$\begin{aligned} n = 3 & \quad \text{the solutions given in Table 1;} \\ n = 4 & \quad \text{the solutions given in Table 2;} \\ n = 5 & \quad (x, y, \alpha, \beta, \gamma) = (401, 11, 1, 3, 0); \\ n = 6 & \quad (x, y, \alpha, \beta, \gamma) = (7, 3, 3, 1, 1), (23, 3, 3, 2, 0); \\ n = 8 & \quad (x, y, \alpha, \beta, \gamma) = (47, 3, 8, 0, 1), (79, 3, 6, 1, 0). \end{aligned}$$

One can deduce from the above result the following corollary.

Corollary 1. *The equation*

$$x^2 + 5^k 17^l = y^n, \quad x \geq 1, \quad y \geq 1, \quad \gcd(x, y) = 1, \quad n \geq 3, \quad k \geq 0, \quad l \geq 0 \quad (3)$$

has only the solutions

$$(x, y, k, l, n) = (94, 21, 2, 1, 3), (2034, 161, 3, 2, 3), (8, 3, 0, 1, 4).$$

Therefore, our work extends that of Pink and Rábai [28]. We will follow the standard approach to work on equation (2) but with another version of MAGMA (V2.18-6) that gives better results when we deal with the corresponding elliptic curves.

2 The case $n = 3$

Lemma 1. *When $n = 3$, all the solutions to equation (2) are given in Table 1.*

For $n = 6$, we have $(x, y, \alpha, \beta, \gamma) = (7, 3, 3, 1, 1), (23, 3, 3, 2, 0)$.

Proof. Equation (2) can be rewritten as

$$\left(\frac{x}{z^3}\right)^2 + A = \left(\frac{y}{z^2}\right)^3, \quad (4)$$

where A is sixth-power free and defined implicitly by $2^\alpha 5^\beta 17^\gamma = Az^6$. One can see that $A = 2^{\alpha_1} 5^{\beta_1} 17^{\gamma_1}$ with $\alpha_1, \beta_1, \gamma_1 \in \{0, 1, 2, 3, 4, 5\}$. We thus get

$$V^2 = U^3 - 2^{\alpha_1} 5^{\beta_1} 17^{\gamma_1}, \quad (5)$$

with $U = y/z^2$, $V = x/z^3$ and $\alpha_1, \beta_1, \gamma_1 \in \{0, 1, 2, 3, 4, 5\}$. We need to determine all the $\{2, 5, 17\}$ -integral points on the above 216 elliptic curves. Recall that if \mathcal{S} is a finite set of prime numbers, then an \mathcal{S} -integer is rational number a/b with coprime integers a and b , where the prime factors of b are in \mathcal{S} . We use the command *SIntegralPoints* of MAGMA [17] to determine all the $\{2, 5, 17\}$ -integer points on the above elliptic curves. Here are a few remarks about the computations:

1. We eliminate the solutions with $UV = 0$ because they yield to $xy = 0$.

Table 1: Solutions for $n = 3$.

| α_1 | β_1 | γ_1 | z | α | β | γ | x | y |
|------------|-----------|------------|-------------|----------|---------|----------|---------------|-----------|
| 1 | 0 | 0 | 1 | 1 | 0 | 0 | 5 | 3 |
| 1 | 0 | 0 | $2 \cdot 5$ | 7 | 6 | 0 | 383 | 129 |
| 2 | 0 | 0 | 1 | 2 | 0 | 0 | 11 | 5 |
| 4 | 0 | 1 | 5 | 4 | 6 | 1 | 5369 | 321 |
| 3 | 0 | 2 | 5 | 3 | 6 | 2 | 167589 | 3041 |
| 1 | 1 | 1 | 2^2 | 13 | 1 | 1 | 93 | 89 |
| 1 | 1 | 1 | 5 | 1 | 7 | 1 | 1531 | 171 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 453 | 59 |
| 3 | 1 | 1 | 1 | 3 | 1 | 1 | 7 | 9 |
| 1 | 1 | 2 | 1 | 1 | 1 | 2 | 63 | 19 |
| 2 | 1 | 2 | 1 | 2 | 1 | 2 | 59 | 21 |
| 1 | 1 | 3 | 2 | 7 | 1 | 3 | 5471 | 321 |
| 1 | 1 | 3 | 5 | 1 | 7 | 3 | 17052501 | 66251 |
| 3 | 2 | 0 | 1 | 3 | 2 | 0 | 23 | 9 |
| 3 | 2 | 0 | 2 | 9 | 2 | 0 | 17771 | 681 |
| 5 | 2 | 0 | 1 | 5 | 2 | 0 | 261 | 41 |
| 0 | 2 | 1 | 1 | 0 | 2 | 1 | 94 | 21 |
| 0 | 2 | 1 | 2 | 6 | 2 | 1 | 55157 | 1449 |
| 3 | 3 | 1 | 2 | 9 | 3 | 1 | 10763 | 489 |
| 3 | 3 | 1 | 2^2 | 15 | 3 | 1 | 4617433 | 27729 |
| 0 | 3 | 2 | 1 | 0 | 3 | 2 | 2034 | 161 |
| 3 | 3 | 5 | 2^5 | 33 | 3 | 5 | 2037783243169 | 160733121 |
| 1 | 4 | 0 | 1 | 1 | 4 | 0 | 9 | 11 |
| 4 | 4 | 1 | $2 \cdot 5$ | 10 | 10 | 1 | 3274947 | 22169 |
| 5 | 4 | 2 | $2 \cdot 5$ | 11 | 10 | 2 | 699659581 | 788121 |
| 1 | 5 | 0 | 17 | 1 | 5 | 6 | 916769 | 9971 |
| 1 | 5 | 1 | 17 | 1 | 5 | 7 | 846227 | 14859 |
| 1 | 5 | 1 | 2 | 7 | 5 | 1 | 17579 | 681 |

2. We consider only solutions such that the numerators of U and V are coprime.
3. If U and V are integers then $z = 1$. So $\alpha_1 = \alpha$, $\beta_1 = \beta$, and $\gamma_1 = \gamma$.
4. If U and V are rational numbers which are not integers, then z is determined by the denominators of U and V . The numerators of these rational numbers give x and y . Then α, β, γ are computed knowing that $2^\alpha 5^\beta 17^\gamma = Az^6$.

Therefore, we first determine $(U, V, \alpha_1, \beta_1, \gamma_1)$ and then we use the relations

$$U = \frac{y}{z^2}, \quad V = \frac{x}{z^3}, \quad 2^\alpha 5^\beta 17^\gamma = Az^6,$$

to find the solutions $(x, y, \alpha, \beta, \gamma)$ listed in Table 1.

For $n = 6$, equation

$$x^2 + 2^\alpha 5^\beta 17^\gamma = y^6 \quad (6)$$

becomes equation

$$x^2 + 2^\alpha 5^\beta 17^\gamma = (y^2)^3. \quad (7)$$

We look in the list of solutions of Table 1 and observe that y is a perfect square only when $y = 9$ corresponding to two solutions. Therefore, the only solutions to equation (2) for $n = 6$ are the two solutions listed in Theorem 1. This completes the proof of Lemma 1. \square

Remark 1. Notice that with the old version of MAGMA, it was difficult to determine the rational points of certain elliptic curves when $2^\alpha 5^\beta 17^\gamma$ is very high. That is the case of the following elliptic curves:

$$V^2 = U^3 - 2^3 \cdot 5^5 \cdot 17^5, \quad V^2 = U^3 - 2^5 \cdot 5^1 \cdot 17^4.$$

We thank the team MAGMA, particularly Steve Donnelly for the new version (Magma V2.18-6) and their help.

3 The case $n = 4$

Here, we have the following result.

Lemma 2. *If $n = 4$, then the only solutions to equation (2) are given in Table 2.*

If $n = 8$, then the only solution to equation (2) is $(x, y, \alpha, \beta, \gamma) = (47, 3, 8, 0, 1)$, $(79, 3, 6, 1, 0)$.

Table 2: Solutions for $n = 4$.

| α_1 | β_1 | γ_1 | z | α | β | γ | x | y |
|------------|-----------|------------|-------|----------|---------|----------|------|-----|
| 1 | 0 | 0 | 2 | 5 | 0 | 0 | 7 | 3 |
| 0 | 1 | 0 | 2 | 4 | 1 | 0 | 1 | 3 |
| 0 | 0 | 1 | 2^2 | 8 | 0 | 1 | 1087 | 33 |
| 0 | 0 | 1 | 1 | 0 | 0 | 1 | 8 | 3 |
| 0 | 0 | 1 | 2^2 | 8 | 0 | 1 | 47 | 9 |
| 1 | 0 | 1 | 2 | 5 | 0 | 1 | 9 | 5 |
| 3 | 0 | 1 | 2 | 7 | 0 | 1 | 15 | 7 |
| 3 | 0 | 1 | 2^2 | 11 | 0 | 1 | 495 | 23 |
| 2 | 1 | 0 | 2 | 6 | 1 | 0 | 79 | 9 |
| 2 | 2 | 1 | 2 | 6 | 2 | 1 | 409 | 21 |
| 3 | 2 | 2 | 2 | 7 | 2 | 2 | 511 | 33 |
| 1 | 0 | 3 | 2^2 | 9 | 0 | 3 | 4785 | 71 |

Proof. Equation (2) can be written as

$$\left(\frac{x}{z^2}\right)^2 + A = \left(\frac{y}{z}\right)^4, \quad (8)$$

where A is fourth-power free and defined implicitly by $2^\alpha 5^\beta 17^\gamma = Az^4$. One can see that $A = 2^{\alpha_1} 5^{\beta_1} 17^{\gamma_1}$ with $\alpha_1, \beta_1, \gamma_1 \in \{0, 1, 2, 3\}$. Hence, the problem consists of determining the $\{2, 5, 17\}$ -integer points on the totality of the 64 elliptic curves

$$V^2 = U^4 - 2^{\alpha_1} 5^{\beta_1} 17^{\gamma_1}, \quad (9)$$

with $U = y/z$, $V = x/z^2$ and $\alpha_1, \beta_1, \gamma_1 \in \{0, 1, 2, 3\}$. Here, we use the command *SIntegralQuarticPoints* of MAGMA [17] to determine the $\{2, 5, 17\}$ -integer points on the above elliptic curves. As in Section 2, we first find $(U, V, \alpha_1, \beta_1, \gamma_1)$, and then using the coprimality conditions on x and y and the definition of U and V , we determine all the corresponding solutions $(x, y, \alpha, \beta, \gamma)$ listed in Table 2.

Looking in the list of solutions of equation Table 2, we observe the 2 solutions in Table 2 whose values for y are perfect squares. Thus, the only solutions to equation (2) with $n = 8$ are those listed in Theorem 1. This concludes the proof of Lemma 2. \square

4 The case $n \geq 5$

The aim of this section is to determine all solutions of equation (2), for $n \geq 5$ and to prove its unsolvability for $n = 7$ and $n \geq 9$. The cases when n is of the form $2^a 3^b$ were treated in previous sections. So, apart from these cases, in order to prove that (2) has no solution for $n \geq 7$, it suffices to consider n prime. In fact, if $(x, y, \alpha, \beta, \gamma, n)$ is a solution for (2) and $n = pk$, where $p \geq 7$ is prime and $k > 1$, then $(x, y^k, \alpha, \beta, \gamma, p)$ is also a solution. So, from now on, n will denote a prime number.

Lemma 3. *The Diophantine equation (2) has no solution with $n \geq 5$ prime except for*

$$n = 5 \quad (x, y, \alpha, \beta, \gamma) = (401, 11, 1, 3, 0).$$

Proof. Let $(x, y, \alpha, \beta, \gamma, n)$ be a solution for (2). We claim that y is odd. In fact, if y is even and since $\gcd(x, y) = 1$, one has that x is odd, and then $-2^\alpha 5^\beta 17^\gamma \equiv x^2 - y^n \equiv 1 \pmod{4}$, but this contradicts the fact that $-2^\alpha 5^\beta 17^\gamma \equiv 0, 2$ or $3 \pmod{4}$ (according to $\alpha \geq 2, \alpha = 1$ or $\alpha = 0$, respectively). Now, write equation (2) as $x^2 + dz^2 = y^n$, where

$$d = 2^{\alpha - 2\lfloor \alpha/2 \rfloor} 5^{\beta - 2\lfloor \beta/2 \rfloor} 17^{\gamma - 2\lfloor \gamma/2 \rfloor},$$

and $z = 2^{\lfloor \alpha/2 \rfloor} 5^{\lfloor \beta/2 \rfloor} 17^{\lfloor \gamma/2 \rfloor}$. Since $x - 2\lfloor x/2 \rfloor \in \{0, 1\}$, we have

$$d \in \{1, 2, 5, 10, 17, 34, 85, 170\}.$$

We then factor the previous equation over $\mathbb{K} = \mathbb{Q}[i\sqrt{d}] = \mathbb{Q}[\sqrt{-d}]$ as

$$(x + i\sqrt{d}z)(x - i\sqrt{d}z) = y^n.$$

Now, we claim that the ideals $(x + i\sqrt{d}z)\mathcal{O}_{\mathbb{K}}$ and $(x - i\sqrt{d}z)\mathcal{O}_{\mathbb{K}}$ are coprime. If this is not the case, there must exist a prime ideal \mathfrak{p} containing these ideals. Therefore, $x \pm i\sqrt{d}z$ and y^n (and so y) belong to \mathfrak{p} . Thus $2x \in \mathfrak{p}$ and hence either 2

or x belongs to \mathfrak{p} . Since $\gcd(2, y) = \gcd(x, y) = 1$, then 1 belongs to the ideals $\langle 2, y \rangle$ and $\langle x, y \rangle$, then $1 \in \mathfrak{p}$ leading to an absurdity of $\mathfrak{p} = \mathcal{O}_{\mathbb{K}}$. By the unique factorization of ideals, it follows that $(x + i\sqrt{d}z)\mathcal{O}_{\mathbb{K}} = \mathfrak{j}^n$, for some ideal \mathfrak{j} of $\mathcal{O}_{\mathbb{K}}$. Using Mathematica's command `NumberFieldClassNumber[Sqrt[-d]]`, we obtain that the class number of \mathbb{K} is either 1, 2, 4 or 12 and so coprime to n , then \mathfrak{j} is a principal ideal yielding

$$x + i\sqrt{d}z = \varepsilon\eta^n, \quad (10)$$

for some $\eta \in \mathcal{O}_{\mathbb{K}}$ and ε a unit of \mathbb{K} . Since the group of units of \mathbb{K} is a subset of $\{\pm 1, \pm i\}$ and n is odd, then ε is a n -th power. Thus, (10) can be reduced to $x + i\sqrt{d}z = \eta^n$. Since \mathbb{K} is an imaginary quadratic field and $-d \not\equiv 1 \pmod{4}$, then $\{1, i\sqrt{d}\}$ is an integral basis and we can write $\eta = u + i\sqrt{d}v$, for some integers u and v . We then get

$$\frac{\eta^n - \bar{\eta}^n}{\eta - \bar{\eta}} = \frac{2^{\lfloor \alpha/2 \rfloor} 5^{\lfloor \beta/2 \rfloor} 17^{\lfloor \gamma/2 \rfloor}}{v}, \quad (11)$$

where, as usual, \bar{w} denotes the complex conjugate of w .

Let $(L_m)_{m \geq 0}$ be the Lucas sequence given by

$$L_m = \frac{\eta^m - \bar{\eta}^m}{\eta - \bar{\eta}}, \quad \text{for } m \geq 0.$$

We recall that the Primitive Divisor Theorem for Lucas sequences ensures for primes $n \geq 5$, that there exists a *primitive divisor* for L_n , except for the finitely many (*defective*) pairs $(\eta, \bar{\eta})$ given in Table 1 of [11] (a primitive divisor of L_n is a prime that divides L_n but does not divide $(\eta - \bar{\eta})^2 \prod_{j=1}^{n-1} L_j$). And a helpful property of a primitive divisor p is that $p \equiv \pm 1 \pmod{n}$.

For $n = 5$, we find in Table 1 in [11] that L_5 has a primitive divisor except for $(u, d, v) = (1, 10, 1)$ which leads to a number $\eta = 1 + i\sqrt{10} \in \mathbb{Q}[i\sqrt{10}]$ ($d = 10$ is one of the possible values of d described in the beginning of this proof), which gives the solution with $n = 5$.

Apart from this case, let p be a primitive divisor of L_n , $n \geq 7$. The identity (11) implies that $p \in \{2, 5, 17\}$ and so $p = 17$, since $p \not\equiv \pm 1 \pmod{n}$, for $p = 2, 5$. Hence, n is a prime dividing 17 ± 1 and so $n = 2$ or 3 which contradicts the fact that $n \geq 7$. This completes the proof of Theorem 1. \square

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