

Editorial

Marcella Palese

It is a pleasure to introduce this issue of *Communications in Mathematics* dedicated to research papers treating various geometric aspects and structures involved in the Calculus of Variations. Most of them were presented in a meeting which obtained the official status of a Satellite Thematic Session of the 6th European Congress of Mathematics held in Krakow, July 2012. The Satellite Thematic Session on *Geometric Methods in Calculus of Variations* was held on the 6th of July at the AGH University of Science and Technology in Krakow, in parallel with the scientific activities of the Congress. Topics covered global analysis, analysis on manifolds, differential geometry, mechanics of particles and systems and general relativity and gravitation.

In the contribution by Fatibene, Francaviglia and Garruto it is shown that, considering the range $3 \leq m \leq 20$, there exist reductive splittings of the spin group only in dimension $m = 4$ (and in this case in any signature), a result which is relevant for applications in loop quantum gravity. In fact, since reductive splittings allow to globally define the standard Barbero-Immirzi connection, in dimension other than 4, for ensuring globality additional structures should be required.

The paper by Francaviglia, myself and Winterroth introduces the concept of conserved current variationally associated with locally variational invariant field equations. It is shown that the invariance of the variation of the corresponding local presentation is a sufficient condition for the current being variationally equivalent to a global one. The case of a Chern-Simons theory is worked out and a global current is variationally associated with a Chern-Simons local Lagrangian.

Havelková considers dynamical properties of singular Lagrangian systems by studying symmetries and conservation laws for a specific singular Lagrangian system of interest in physics. It is investigated whether to every point symmetry of a Euler-Lagrange form there exists a Lagrangian such that the symmetry is also a point symmetry of the Lagrangian itself. For the system under consideration the answer is affirmative and the corresponding Lagrangians are all of order one.

Muzsnay and Nagy characterized the 3-dimensional Heisenberg group with left invariant cubic metric as an example of Finsler manifold having infinite dimensional curvature algebra and holonomy group. The aim of their paper is to describe the algebraic structure of this curvature algebra; they prove that it is an infinite

dimensional graded Lie subalgebra of the generalized Witt algebra of homogeneous vector fields generated by three elements.

The paper by Rossi and Musilová treats an important aspect of the inverse problem of the calculus of variations in a nonholonomic setting. The concept of constraint variationality is introduced in the context of first order mechanical systems with general nonholonomic constraints and it is shown that such a concept is equivalent with the existence of a closed representative in the class of 2-forms determining the nonholonomic system. Together with constraint Helmholtz conditions this result completes the basic geometric properties of constraint variational systems.

Saunders deals with the projective Finsler metrizable problem, precisely with the question whether a projective-equivalence class of sprays is the geodesic class of a (locally or globally defined) Finsler function; this paper reviews an interesting approach to the problem using an analogue of the multiplier approach to the inverse problem in Lagrangian mechanics. Conditions are determined for the existence of a global pseudo-Finsler function with Euler-Lagrange equations satisfied by the geodesics of the sprays.

Our meeting enjoyed a pleasant, friendly and stimulating atmosphere promoting interactions between various aspects and topics in the Calculus of Variations. My thanks to all who contributed to this intent and, particularly, to Prof. Olga Rossi for her fundamental help in the organization and the successful outcome of this event.

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Marcella Palese
Guest Editor

Editor's address:

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF TORINO, VIA C. ALBERTO 10, I-10123 TORINO,
ITALY

E-mail: marcella.palese@unito.it