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Distinguished Riemann-Hamilton geometry in the polymomentum electrodynamics

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Abstract. In this paper we develop the distinguished (d-) Riemannian differential geometry (in the sense of d-connections, d-torsions, d-curvatures and some geometrical Maxwell-like and Einstein-like equations) for the polymomentum Hamiltonian which governs the multi-time electrodynamics.

1 Introduction

Let M^n be a smooth real manifold of dimension n , whose local coordinates are $x = (x^i)_{i=1,n}$, having the physical meaning of “space of events”. In order to justify the “electrodynamics” terminology used in this paper, we recall that, in the study of classical electrodynamics, the Lagrangian function $L: TM \rightarrow \mathbb{R}$ that governs the movement law of a particule of mass $m \neq 0$ and electric charge e , placed concomitantly into a gravitational field and an electromagnetic one, is expressed by

$$L(x, y) = mc\varphi_{ij}(x)y^i y^j + \frac{2e}{m}A_i(x)y^i + \mathcal{P}(x), \quad (1)$$

where the semi-Riemannian metric $\varphi_{ij}(x)$ represents the *gravitational potentials* of the space M , $A_i(x)$ are the components of an 1-form on M representing the *electromagnetic potential*, $\mathcal{P}(x)$ is a smooth *potential function* on M and c is the velocity of light in vacuum. The Lagrange space $L^n = (M, L(x, y))$, where L is given by (1), is known in the literature of specialty as the *autonomous Lagrange space of electrodynamics*. A deep geometrical study of the Lagrange space L^n is now completely done in Miron-Anastasiu's book [15]. More general, in the study of classical *time-dependent* electrodynamics, a central role is played by the

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autonomous time-dependent Lagrangian function of electrodynamics:

$$L(t, x, y) = mc\varphi_{ij}(x)y^iy^j + \frac{2e}{m}A_i(t, x)y^i + \mathcal{P}(t, x), \quad (2)$$

where $L: \mathbb{R} \times TM \rightarrow \mathbb{R}$. Note that the non-dynamical character (i.e., the independence on the temporal coordinate t) of the spatial semi-Riemannian metric $\varphi_{ij}(x)$ determines the usage of the term “*autonomous*” in the preceding definition.

Let $(\mathcal{T}^m, h_{ab}(t))$ be a “*multi-time*” smooth Riemannian manifold of dimension m (please do not confuse with the mass $m \neq 0$), having the local coordinates $t = (t^c)_{c=1, \dots, m}$, and let $J^1(\mathcal{T}, M)$ be the 1-jet space produced by the manifolds \mathcal{T} and M .

Remark 1. The use in our work of the “*multi-time*” terminology was lent by us from Dickey’s monograph [6]. However, it is important to note that “*multi-time*” does not mean a “*multidimensional time*”, but has the sense of a “*multi-parameter*” or “*many parameters*”.

By a natural extension of the preceding examples of electrodynamics Lagrangian functions, we can consider the jet multi-time Lagrangian function

$$L(t^c, x^k, x_c^k) = mch^{ab}(t)\varphi_{ij}(x)x_a^ix_b^j + \frac{2e}{m}A_{(i)}^{(a)}(t, x)x_a^i + \mathcal{P}(t, x), \quad (3)$$

where $A_{(i)}^{(a)}(t, x)$ is a d-tensor on $J^1(\mathcal{T}, M)$ and $\mathcal{P}(t, x)$ is a smooth function on the product manifold $\mathcal{T} \times M$.

Remark 2. Throughout this paper, the indices a, b, c, \dots run from 1 to m , while the indices i, j, k, \dots run from 1 to n . The Einstein convention of summation is also adopted all over this work.

The pair $\mathcal{EDML}_m^n = (J^1(\mathcal{T}, M), L)$, where L is given by (3), is called the *autonomous multi-time Lagrange space of electrodynamics*. The distinguished Riemannian geometrization of the multi-time Lagrange space \mathcal{EDML}_m^n is now completely developed in the Neagu’s works [17] and [18].

Via the classical Legendre transformation, the jet multi-time Lagrangian function of electrodynamics (3) leads us to the Hamiltonian function of polymomenta

$$H = \frac{1}{4mc}h_{ab}\varphi^{ij}p_i^ap_j^b - \frac{e}{m^2c}h_{ab}\varphi^{ij}A_{(j)}^{(b)}p_i^a + \frac{e^2}{m^3c}\|A\|^2 - \mathcal{P}, \quad (4)$$

where $H: J^{1*}(\mathcal{T}, M) \rightarrow \mathbb{R}$, and

$$\|A\|^2(t, x) = h_{ab}\varphi^{ij}A_{(i)}^{(a)}A_{(j)}^{(b)}.$$

Definition 1. The pair $\mathcal{EDMH}_m^n = (J^{1*}(\mathcal{T}, M), H)$, where H is given by (4), is called the *autonomous multi-time Hamilton space of electrodynamics*.

But, using as a pattern the Miron's geometrical ideas from [16], the distinguished Riemannian geometry for quadratic Hamiltonians of polymomenta (geometry in the sense of d-connections, d-torsions, d-curvatures and geometrical Maxwell-like and Einstein-like equations) is constructed on dual 1-jet spaces in the Oană-Neagu's paper [21]. Consequently, in what follows, we apply the general geometrical result from [21] for the particular Hamiltonian function of polymomenta (4), which governs the multi-time electrodynamics.

2 The geometry of the autonomous multi-time Hamilton space of electrodynamics \mathcal{EDMH}_m^n

To initiate our Hamiltonian geometrical development for multi-time electrodynamics, let us consider on the dual 1-jet space $E^* = J^{1*}(\mathcal{T}, M)$ the *fundamental vertical metrical d-tensor*

$$\Phi_{(a)(b)}^{(i)(j)} = \frac{1}{2} \frac{\partial^2 H}{\partial p_i^a \partial p_j^b} = h_{ab}^*(t^c) \varphi^{ij}(x^k),$$

where $h_{ab}^*(t) := (4mc)^{-1} \cdot h_{ab}(t)$. Let $\chi_{bc}^a(t)$ (respectively $\gamma_{ij}^k(x)$) be the Christoffel symbols of the metric $h_{ab}(t)$ (respectively $\varphi_{ij}(x)$). Obviously, if $\overset{*}{\chi}_{bc}^a$ are the Christoffel symbols of the Riemannian metric $h_{ab}^*(t)$, then we have $\overset{*}{\chi}_{bc}^a = \chi_{bc}^a$.

Using a general result from the geometrical theory of multi-time Hamilton spaces (see [2] and [21]), by direct computations, we find

Theorem 1. *The pair of local functions $N_{\mathcal{ED}} = \left(N_{1(i)b}^{(a)}, N_{2(i)j}^{(a)} \right)$ on the dual 1-jet space E^* , which are given by*

$$N_{1(i)b}^{(a)} = \chi_{bf}^a p_i^f, \quad N_{2(i)j}^{(a)} = \gamma_{ij}^r \left[\frac{2e}{m} A_{(r)}^{(a)} - p_r^a \right] - \frac{e}{m} \left[\frac{\partial A_{(i)}^{(a)}}{\partial x^j} + \frac{\partial A_{(j)}^{(a)}}{\partial x^i} \right],$$

represents a nonlinear connection on E^* . This nonlinear connection is called the canonical nonlinear connection of the multi-time Hamilton space of electrodynamics \mathcal{EDMH}_m^n .

Now, let

$$\left\{ \frac{\delta}{\delta t^a}, \frac{\delta}{\delta x^i}, \frac{\partial}{\partial p_i^a} \right\} \subset \chi(E^*), \quad \{ dt^a, dx^i, \delta p_i^a \} \subset \chi^*(E^*)$$

be the adapted bases produced by the nonlinear connection $N_{\mathcal{ED}}$, where

$$\begin{aligned} \frac{\delta}{\delta t^a} &= \frac{\partial}{\partial t^a} - N_{1(r)a}^{(f)} \frac{\partial}{\partial p_r^f}, & \frac{\delta}{\delta x^i} &= \frac{\partial}{\partial x^i} - N_{2(r)i}^{(f)} \frac{\partial}{\partial p_r^f}, \\ \delta p_i^a &= dp_i^a + N_{1(i)f}^{(a)} dt^f + N_{2(i)r}^{(a)} dx^r. \end{aligned} \quad (5)$$

Working with these adapted bases, by direct computations, we can determine the adapted components of the *generalized Cartan canonical connection* of the space \mathcal{EDMH}_m^n , together with its local d-torsions and d-curvatures (for details, see the general formulas from [21]).

Theorem 2. (1) The generalized Cartan canonical linear connection of the autonomous multi-time Hamilton space of electrodynamics \mathcal{EDMH}_m^n is given by

$$C\Gamma(N) = \left(\chi_{bc}^a, A_{jc}^i, H_{jk}^i, C_{j(c)}^{i(k)} \right),$$

where its adapted components are

$$H_{ab}^c = \chi_{ab}^c, \quad A_{jc}^i = 0, \quad H_{jk}^i = \gamma_{jk}^i, \quad C_{j(c)}^{i(k)} = 0. \quad (6)$$

(2) The torsion \mathbb{T} of the generalized Cartan canonical linear connection of the space \mathcal{EDMH}_m^n is determined by three effective adapted components:

$$\begin{aligned} R_{(r)ab}^{(f)} &= \chi_{gab}^f p_r^g, \\ R_{(r)aj}^{(f)} &= -\frac{2e}{m} \gamma_{rj}^s A_{(s);a}^{(f)} + \frac{e}{m} \left[\frac{\partial A_{(r)}^{(f)}}{\partial x^j} + \frac{\partial A_{(j)}^{(f)}}{\partial x^r} \right]_{;a}, \\ R_{(r)ij}^{(f)} &= \mathfrak{R}_{rij}^s \left[\frac{2e}{m} A_{(s)}^{(f)} - p_s^f \right] - \frac{e}{m} \left[\frac{\partial A_{(i)}^{(f)}}{\partial x^j} - \frac{\partial A_{(j)}^{(f)}}{\partial x^i} \right]_{;r}, \end{aligned} \quad (7)$$

where $\chi_{dab}^c(t)$ (respectively $\mathfrak{R}_{rij}^k(x)$) are the classical local curvature tensors of the Riemannian metric $h_{ab}(t)$ (respectively semi-Riemannian metric $\varphi_{ij}(x)$), and “ $_{;a}$ ” and “ $_{;k}$ ” represent the following generalized Levi-Civita covariant derivatives:

- the \mathcal{T} -generalized Levi-Civita covariant derivative:

$$\begin{aligned} T_{cj(l)(f)\dots;a}^{bi(d)(r)\dots} &\stackrel{\text{def}}{=} \frac{\partial T_{cj(l)(f)\dots}^{bi(d)(r)\dots}}{\partial t^a} + T_{cj(l)(f)\dots}^{gi(d)(r)\dots} \chi_{ga}^b + T_{cj(l)(f)\dots}^{bi(g)(r)\dots} \chi_{ga}^d \\ &\quad + \dots - T_{gj(l)(f)\dots}^{bi(d)(r)\dots} \chi_{ca}^g - T_{cj(l)(g)\dots}^{bi(d)(r)\dots} \chi_{fa}^g - \dots, \end{aligned}$$

- the \mathcal{M} -generalized Levi-Civita covariant derivative:

$$\begin{aligned} T_{cj(l)(f)\dots;k}^{bi(d)(r)\dots} &\stackrel{\text{def}}{=} \frac{\partial T_{cj(l)(f)\dots}^{bi(d)(r)\dots}}{\partial x^k} + T_{cj(l)(f)\dots}^{bs(d)(r)\dots} \gamma_{sk}^i + T_{cj(l)(f)\dots}^{bi(d)(s)\dots} \gamma_{sk}^r \\ &\quad + \dots - T_{cs(l)(f)\dots}^{bi(d)(r)\dots} \gamma_{jk}^s - T_{cj(s)(f)\dots}^{bi(d)(r)\dots} \gamma_{lk}^s - \dots. \end{aligned}$$

(3) The curvature \mathbb{R} of the Cartan canonical connection of the space \mathcal{EDMH}_m^n is determined by the following four effective adapted components:

$$H_{abc}^d = \chi_{abc}^d, \quad R_{ijk}^l = \mathfrak{R}_{ijk}^l$$

and

$$-R_{(l)(a)bc}^{(d)(i)} = \delta_l^i \chi_{abc}^d, \quad -R_{(i)(a)jk}^{(d)(l)} = -\delta_a^d \mathfrak{R}_{ijk}^l.$$

3 Electromagnetic-like model on the multi-time Hamilton space of electrodynamics \mathcal{EDMH}_m^n

In order to describe our geometrical electromagnetic-like theory (depending on polymomenta) on the multi-time Hamilton space of electrodynamics \mathcal{EDMH}_m^n , we underline that, by a simple direct calculation, we obtain (see [21]).

Proposition 1. *The metrical deflection d-tensors of the space \mathcal{EDMH}_m^n are expressed by the formulas:*

$$\begin{aligned}\Delta_{(a)b}^{(i)} &= \left[h_{af}^* \varphi^{ir} p_r^f \right]_{/b} = 0, & \vartheta_{(a)(b)}^{(i)(j)} &= \left[h_{af}^* \varphi^{ir} p_r^f \right]_{(b)}^{(j)} = \frac{1}{4mc} h_{ab} \varphi^{ij}, \\ \Delta_{(a)j}^{(i)} &= \left[h_{af}^* \varphi^{ir} p_r^f \right]_{|j} = \frac{e}{4m^2 c} h_{af} \varphi^{ir} \left[A_{(r):j}^{(f)} + A_{(j):r}^{(f)} \right],\end{aligned}\quad (8)$$

where “ $/b$ ”, “ $|j$ ” and “ $\left. \vphantom{\Delta} \right|_{(j)}^{(b)}$ ” are the local covariant derivatives induced by the generalized Cartan canonical connection $CT(N)$ (see [20] and [21]).

Moreover, taking into account some general formulas from [21], we introduce

Definition 2. The distinguished 2-form on $J^{1*}(\mathcal{T}, M)$, locally defined by

$$\mathbb{F} = F_{(a)j}^{(i)} \delta p_i^a \wedge dx^j + f_{(a)(b)}^{(i)(j)} \delta p_i^a \wedge \delta p_j^b, \quad (9)$$

where

$$\begin{aligned}F_{(a)j}^{(i)} &= \frac{1}{2} \left[\Delta_{(a)j}^{(i)} - \Delta_{(a)i}^{(j)} \right] = \frac{e}{8m^2 c} \cdot \mathcal{A}_{\{i,j\}} \left\{ h_{af} \varphi^{ir} \left[A_{(r):j}^{(f)} + A_{(j):r}^{(f)} \right] \right\}, \\ f_{(a)(b)}^{(i)(j)} &= \frac{1}{2} \left[\vartheta_{(a)(b)}^{(i)(j)} - \vartheta_{(a)(b)}^{(j)(i)} \right] = 0,\end{aligned}\quad (10)$$

is called the *polymomentum electromagnetic field attached to the multi-time Hamilton space of electrodynamics \mathcal{EDMH}_m^n* .

Now, particularizing the generalized Maxwell-like equations of the polymomentum electromagnetic field that govern a general multi-time Hamilton space MH_m^n , we obtain the main result of the polymomentum electromagnetism on the space \mathcal{EDMH}_m^n (for more details, see [21]):

Theorem 3. *The polymomentum electromagnetic components (10) of the autonomous multi-time Hamilton space of electrodynamics \mathcal{EDMH}_m^n are governed by*

the following geometrical Maxwell-like equations:

$$\left\{ \begin{array}{l} F_{(a)j/b}^{(i)} = F_{(a)j;b}^{(i)} = \frac{e \cdot h_{af}}{8m^2 c} \cdot \mathcal{A}_{\{i,j\}} \left\{ \varphi^{ir} \left[\frac{\partial A_{(r)}^{(f)}}{\partial x^j} + \frac{\partial A_{(j)}^{(f)}}{\partial x^r} \right] - 2\varphi^{ir} \gamma_{rj}^s A_{(s);b}^{(f)} \right\} \\ \sum_{\{i,j,k\}} F_{(a)j|k}^{(i)} = \sum_{\{i,j,k\}} F_{(a)j:k}^{(i)} = -\frac{h_{af}}{8mc} \cdot \sum_{\{i,j,k\}} \left\{ \left[\varphi^{sr} \mathfrak{R}_{rjk}^i - \varphi^{ir} \mathfrak{R}_{rjk}^s \right] p_s^f + \right. \\ \left. + \frac{e}{m} \varphi^{ir} \left[2\mathfrak{R}_{rjk}^s A_{(s)}^{(f)} - \left(\frac{\partial A_{(j)}^{(f)}}{\partial x^k} - \frac{\partial A_{(k)}^{(f)}}{\partial x^j} \right) \right] \right\} \\ \sum_{\{i,j,k\}} F_{(a)j|^{(k)}(c)}^{(i)} = 0, \end{array} \right. \quad (11)$$

where $\mathcal{A}_{\{i,j\}}$ represents an alternate sum, $\sum_{\{i,j,k\}}$ represents a cyclic sum, and we have

$$F_{(a)j|^{(k)}(c)}^{(i)} = \frac{\partial F_{(a)j}^{(i)}}{\partial p_k^c} = 0.$$

4 Gravitational-like geometrical model on the multi-time Hamilton space of electrodynamics

To expose our geometrical Hamiltonian polymomentum gravitational theory on the autonomous multi-time Hamilton space of electrodynamics \mathcal{EDMH}_m^n , we recall that the fundamental vertical metrical d-tensor

$$\Phi_{(a)(b)}^{(i)(j)} = h_{ab}^*(t) \varphi^{ij}(x)$$

and the canonical nonlinear connection

$$N_{\mathcal{ED}} = \left(N_{1(i)b}^{(a)}, N_{2(i)j}^{(a)} \right)$$

of the multi-time Hamilton space \mathcal{EDMH}_m^n produce a polymomentum gravitational h^* -potential \mathbb{G} on $E^* = J^{1*}(\mathcal{T}, M)$, locally expressed by

$$\mathbb{G} = h_{ab}^* dt^a \otimes dt^b + \varphi_{ij} dx^i \otimes dx^j + h_{ab}^* \varphi^{ij} \delta p_i^a \otimes \delta p_j^b. \quad (12)$$

We postulate that the *geometrical Einstein-like equations*, which govern the multi-time gravitational h^* -potential \mathbb{G} of the multi-time Hamilton space of electrodynamics \mathcal{EDMH}_m^n , are the abstract geometrical Einstein equations attached to the Cartan canonical connection $CT(N)$ and to the adapted metric \mathbb{G} on E^* , namely

$$\text{Ric}(CT) - \frac{\text{Sc}(CT)}{2} \mathbb{G} = \mathcal{K} \mathbb{T}, \quad (13)$$

where $\text{Ric}(CT)$ represents the *Ricci tensor* of the Cartan connection, $\text{Sc}(CT)$ is the *scalar curvature*, \mathcal{K} is the *Einstein constant* and \mathbb{T} is an intrinsic d-tensor of matter, which is called the *stress-energy d-tensor of polymomenta*.

In order to describe the local geometrical Einstein-like equations (together with their generalized conservation laws) in the adapted basis

$$\{X_A\} = \left\{ \frac{\delta}{\delta t^a}, \frac{\delta}{\delta x^i}, \frac{\partial}{\partial p_i^a} \right\},$$

let $CT(N) = (\chi_{ab}^c, 0, \gamma_{jk}^i, 0)$ be the generalized Cartan canonical connection of the space \mathcal{EDMH}_m^n . Taking into account the expressions of its adapted curvature d-tensors on the space \mathcal{EDMH}_m^n , we immediately find (see [21]):

Theorem 4. *The Ricci tensor $\text{Ric}(CT)$ of the autonomous multi-time Hamilton space of electrodynamics \mathcal{EDMH}_m^n is characterized by two effective local Ricci d-tensors:*

$$\chi_{ab} = \chi_{abf}^f, \quad \mathfrak{R}_{ij} = \mathfrak{R}_{ijr}^r.$$

These are exactly the classical Ricci tensors of the Riemannian temporal metric $h_{ab}(t)$ and the semi-Riemannian spatial metric $\varphi_{ij}(x)$.

Consequently, using the notations $\chi = h^{ab}\chi_{ab}$ and $\mathfrak{R} = \varphi^{ij}\mathfrak{R}_{ij}$, we get

Theorem 5. *The scalar curvature $\text{Sc}(CT)$ of the generalized Cartan connection CT of the space \mathcal{EDMH}_m^n has the expression (for details, see [21])*

$$\text{Sc}(CT) = (4mc) \cdot \chi + \mathfrak{R},$$

where χ and \mathfrak{R} are the classical scalar curvatures of the semi-Riemannian metrics $h_{ab}(t)$ and $\varphi_{ij}(x)$.

Particularizing the generalized Einstein-like equations and the generalized conservation laws of an arbitrary multi-time Hamilton space MH_m^n , we can establish the main result of the geometrical polymomentum gravitational theory on the autonomous multi-time Hamilton space of electrodynamics \mathcal{EDMH}_m^n (for more details, see [21]):

Theorem 6. (1) *The local geometrical Einstein-like equations, that govern the polymomentum gravitational potential of the space \mathcal{EDMH}_m^n , have the form*

$$\left\{ \begin{array}{l} \chi_{ab} - \frac{(4mc) \cdot \chi + \mathfrak{R}}{8mc} h_{ab} = \mathcal{K}\mathbb{T}_{ab} \\ \mathfrak{R}_{ij} - \frac{(4mc) \cdot \chi + \mathfrak{R}}{2} \varphi_{ij} = \mathcal{K}\mathbb{T}_{ij} \\ -\frac{(4mc) \cdot \chi + \mathfrak{R}}{8mc} h_{ab} \varphi^{ij} = \mathcal{K}\mathbb{T}_{(a)(b)}^{(i)(j)}, \end{array} \right. \quad (14)$$

$$\left\{ \begin{array}{l} 0 = \mathbb{T}_{ai}, \quad 0 = \mathbb{T}_{ia}, \quad 0 = \mathbb{T}_{(a)b}^{(i)} \\ 0 = \mathbb{T}_{a(b)}^{(j)}, \quad 0 = \mathbb{T}_{i(b)}^{(j)}, \quad 0 = \mathbb{T}_{(a)j}^{(i)}, \end{array} \right. \quad (15)$$

where \mathbb{T}_{AB} , $A, B \in \left\{ a, i, \binom{i}{a} \right\}$, are the adapted components of the polymomentum stress-energy d-tensor of matter \mathbb{T} .

- (2) The polymomentum conservation laws of the geometrical Einstein-like equations of the space \mathcal{EDMH}_m^n are expressed by the formulas

$$\left\{ \begin{array}{l} \left[(4mc) \cdot \chi_b^f - \frac{(4mc) \cdot \chi + \mathfrak{R}}{2} \delta_b^f \right]_{/f} = 0 \\ \left[\mathfrak{R}_j^r - \frac{(4mc) \cdot \chi + \mathfrak{R}}{2} \delta_j^r \right]_{|r} = 0, \end{array} \right. \quad (16)$$

where $\chi_b^f = h^{fd} \chi_{db}$ and $\mathfrak{R}_j^r = \varphi^{rs} \mathfrak{R}_{sj}$.

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