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Book Review

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Mathematical results in quantum physics. Edited by P. Exner. World Scientific, Singapore (2011), 274 pages, ISBN 978-981-4350-35-8

In 1980s a group of Czech mathematical physicists worked at Dubna. In 1987, a distinguished member of the group Pavel Exner together with his collaborator Petr Šeba organized there a conference on mathematical physics oriented especially to the Schrödinger operators with contact interactions. After its success the conference has been repeated and its scope has enlarged. It has moved to other places and also other groups take part in the organization. The name established as "Mathematical Results in Quantum Physics" and the acronym as QMath. On the 6–10 September 2010, QMath11 took place at Hradec Králové in the Czech Republic. 130 participant affiliated in 22 countries registered at the conference.

The Proceedings contain contributions based on the most Plenary Talks and the Invited Talks at the topical sessions. Abstracts of the other invited as well as of the contributed talks are included. A DVD with presentations of most of the talks delivered to the conference is attached for the convenience of the reader. It contains also some photographs illustrating the atmosphere. The published plenary talks are the following.

N. Datta: Relative entropies and entanglement monotones, giving the two new definitions of relative entropies and showing their use in the quantum information theory. The description of quantum states by density matrix operator on a finite dimensional Hilbert space is used.

R.L Frank, E.H. Lieb, R. Seiringer, L.E. Thomas: Binding, stability, and non-binding of multi-polaron systems. Polaron is an object consisting of an electron moving in a crystal and interacting with the crystal lattice excitations modeled by a quantized boson field. Electron is assumed localized in a region large with respect to crystal lattice spacing so it can be assumed as moving in continuum.

The authors consider a model Hamiltonian for the system of N polarons

$$
\begin{aligned}
H_U^{(N)} = & \sum_{j=1}^{N} \mathbf{p}_j^2 + \int a^\dagger(\mathbf{k}) a(\mathbf{k}) d\mathbf{k} \\
& + \frac{\sqrt{\alpha}}{\sqrt{2}\pi} \sum_{j=1}^{N} \int \frac{1}{|k|} [a(\mathbf{k}) \exp(i\mathbf{k}\cdot\mathbf{x}_j) + a^\dagger(\mathbf{k}) \exp(-i\mathbf{k}\cdot\mathbf{x}_j)] d\mathbf{k} \\
& + U \sum_{1 \le i < j \le N} |\mathbf{x}_i - \mathbf{x}_j|^{-1} .
\end{aligned}
$$

Here $\mathbf{x}_j$ are coordinates of the electrons, $p_j = -i\nabla_j$ their momenta, a and $a^\dagger$ bosonic field annihilation and creation operators. Constants α and U are parameters of the model, some other constants are simplified by the choice of units. For the three-dimensional case, the Hamiltonian acts in the Hilbert space $L^2(\mathbb{R}^{3N}) \otimes \mathcal{F}$ where $\mathcal{F}$ is the Fock space of the boson field. The infimum $E_U^{(N)}(\alpha)$ of the $H_U^{(N)}$ spectrum is not an eigenvalue. The existence of binding, i.e. the relation between $E_U^{(N)}(\alpha)$ and $NE_U^{(1)}(\alpha)$, is studied for some ranges of parameters. The authors give up to date review of the known results and their own recent results. Thermodynamic stability, $N^{-1}E_U^{(N)}(\alpha) \ge -\mathit{constant}$ independent of N, holds for $U > 2\alpha > 0$.

A. Giuliani: Interacting electrons on the honeycomb lattice. As a model of graphene, two-dimensional one-atom thick graphite layer, a Hubbard model on a honeycomb lattice is considered. It describes hopping of electrons between lattice vertices. Interaction with the three-dimensional electromagnetic field is further introduced. Mainly the properties of ground state are discussed.

M. Lewin: Renormalization of Dirac's polarized vacuum. A mean field theory for the electrons in a atom or molecule is developed. The Hamiltonian of the Dirac equation with the self-consistent Coulomb field generated by the atomic nuclei, finite number of real electrons and virtual electrons of the Dirac sea is studied. Its spectral projection to the energies below the Fermi level is looked for. Its existence can be proved under an ultraviolet cut-off only. Renormalization of the charge is discussed. The existence of the asymptotic expansion in the renormalized coupling constant for the renormalized nuclear charge density is shown.

O. Post: Convergence result for thick graphs. The problem of approximation of the Laplacian spectral properties on a domain X_ε containing a graph X_0 by that on the graph itself (with an appropriate boundary conditions at the vertices) is discussed. The domain is assumed to be in a sense close to the graph, shrinking to the graph if a small parameter ε approaches zero. Its geometry is explained, especially the shape of neighborhoods of vertices and edges. The two Laplace-like non-negative operators H_ε and H_0 are defined in different Hilbert spaces $\mathcal{H}_\varepsilon = L^2(X_\varepsilon)$ and $\mathcal{H}_0 = L^2(X_0)$. A linear bounded operator $J : \mathcal{H}_0 \to \mathcal{H}_\varepsilon$ is needed for their comparison. Typically, the range of J contains transversally constant functions. It is said that $H_\varepsilon \to H_0$ in the generalized norm resolvent sense of order $\mathcal{O}(\varepsilon^{1/2})$ if and only if there exists J such that

$$
J^*J = \mathrm{id}_0 , \quad \|(\mathrm{id}_\varepsilon - JJ^*)R_\varepsilon\| = \mathcal{O}(\varepsilon^{1/2}) , \quad \|JR_0 - R_\varepsilon J\| = \mathcal{O}(\varepsilon^{1/2})
$$

where $R_\varepsilon = (H_\varepsilon + 1)^{-1}$ denotes the resolvent. Some conditions sufficient for the validity of the last relation are given and the consequences for the spectra are discussed. In the Dirichlet case, the first eigenvalue of the transverse Laplacian diverging as $\varepsilon \to 0$ must be subtracted from the operator of course.

B. Schlein: Spectral properties of Wigner matrices. Hermitian Wigner matrices are finite $N \times N$ Hermitian matrices, the entries of which are random variables, up to the hermiticity independent, with the same distribution law for the diagonal entries and the real and the imaginary parts of the nondiagonal entries. Wigner introduced these matrices as a model of heavy nuclei, they are useful as models of other complex or chaotic systems as well. A review of know spectral properties is given and the new author's result on the statistics of the spectrum is formulated.

R. Sims: Lieb-Robinson bounds and quasi-locality for the dynamics of many-body quantum systems. Roughly speaking, the velocity of disturbances propagation in a lattice of quantum systems is studied.

M. Aizenman, S. Warzel: Disorder-induced delocalization on tree graphs. Random Schrödinger operator on a regular tree graph is shown to have absolutely continuous spectrum in a suitable regime.

T. Weidl: Semiclassical spectral bounds and beyond. The Schrödinger like operator

$$H(V) = (-\Delta)^l - V(x)\,, \quad l > 0\,, \quad V(x) \geq 0\,, \quad x \in \mathbb{R}^d$$

in $L^2(\mathbb{R}^d)$ is considered and the sum

$$S_{d,\gamma}(V) = \sum_j \lambda_j^\gamma = \mathrm{Tr}(H(V))_-^\gamma\,, \quad \gamma \geq 0$$

is defined where $-\lambda_j$ are negative eigenvalues of $H(V)$. The ranges of validity of Lieb-Thirring estimates

$$S_{d,\gamma}(V) \leq R(d,\gamma,l) S_{d,\gamma}^{cl}(V)$$

are discussed where classical phase space average

$$S_{d,\gamma}^{cl}(V) = \iint_{\mathbb{R}^d \times \mathbb{R}^d} (|\xi|^{2l} - V(x))_- \frac{dx\, d\xi}{(2\pi)^d}\,.$$

Possible values of constants $R(d,\gamma,l)$ are studied. Similar estimate from below and several generalizations of the quantity $S_{d,\gamma}(V)$, also in a subdomain of $\mathbb{R}^d$, are investigated.

Among seven topical sessions the one devoted to the honour of Ari Laptev, the president of European Mathematical Society in 2007-2010, on the occasion of his sixtieth birthday had a special significance. Contributions on the talks of *R.D. Benguria* on spectral problems in spaces of constant curvature, *R.L. Frank and L. Geisinger* on the two-term spectral asymptotic for the Dirichlet Laplacian, *B. Helffer* on the Ginzburg-Landau functional, *V. Dinu, A. Jensen and G. Nenciu* on the resonance decay law, and *M. Loss and G. Stolz* on the localization for the random displacement model are included in the proceedings.

More evolved contributions from the other topical sessions are those of *J. Dolbeault and M.J. Esteban*, and *J. Lampart, S. Teufel and J. Wachsmuth* on the spectral theory, *W.D. Roeck*, *D. Hasler and I. Herbst*, and *G.A. Hagedorn and A. Joye* on the many-body quantum systems, *L.F. Santos and M. Rigol*, and *S. Nonnenmacher* on the quantum chaos, *M. Fiałkowski, A. Bitner and R. Hołyst*, *B. Steffen, A. Seyfried and M. Boltes*, and *D. Vašata, P. Exner and P. Šeba* on the physics of social systems.

As most of other proceedings, the book can be recommended to the reader who is looking for a brief, still up to date, information on the above mentioned topics with the references to detailed proofs.

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